

Mechanical Modeling of a Solar Water Geyser

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Abstract: The purpose of this research paper is to develop an understanding that how to build a solar water geyser (SWG) that provides hot water during winter season and distilled water during summer. Calculations were the initial steps to determine the area, size and capacity of storage tank. The amount of solar radiations falling in different areas and different seasons is different. Therefore after calculations, the physical model may have slight differences from theoretical results. The solar geyser used is passive type because there is no external pump and flow is due to gravity and thermo-siphoning. The inclination angle of the solar collector can be changed as it is made adjustable by using a new concept of screw jack as used in raising cars and trucks. The solar geyser consists of copper tubes painted dull black because of the fact that black color absorbs maximum amount of radiations. The solar energy is allowed to be entrapped in the collector as it is laminated with glass and convection of cold air is prevented and the warm water is not allowed to lose most of its thermal energy, this procedure is also called “green house effect”. There is a black flat plate so the maximum sunlight can be absorbed by the back side of the tubes also. The storage tank has a capacity of 80 liters which is almost satisfactory to fulfill the demand of one family. There is also a calculated thermal insulation of the tank that prevents hot water from becoming cold at nights or low temperatures. Therefore there is no consumption of either electricity or natural gas. This geyser can be used as an economical mass product and installed in domestic as well as different industries also. As solar geyser needs very little or no maintenance, therefore it could save economy of a nation to a great extent.

Keywords: Solar Geysers, Mathematical Modeling Black Body Radiation, Green House effect, Thermal energy, Thermal Insulation

I. INTRODUCTION

Solar energy radiation light and heat from the sun has been harnessed by human since ancient times a range of ever evolving technologies. Solar energy technologies include solar heating, solar thermal electricity and solar photovoltaic which can make considerable contributions to solving some of the most urgent problems the world now faces.

Solar technologies are broadly characterized as either passive solar or active solar depending on the way they captured convert and distribute solar energy.

A. Thermosiphoning (The Principle Of Heating Water) [1]

The thermosiphon water system is a simple, passive system i-e there is no pump used. For storing water overnight or on cloudy days, a storage tank is needed. A very simple way of doing this, making use of gravity is the thermosiphon system.

The principle of the thermosiphon system is that cold water has a higher specific density than warm water, and so being heavier will sink down. Therefore, the collector is always mounted below the water storage tank, so that cold water from the tank reaches the collector by moving down to water pipe. If the collector heats up the water, the water rises again and reaches the tank through an upward movement in water pipe at the upper end of the collector. The cycle of tank–water pipe–collector

ensures the water is heated up until it achieves an Equation equilibrium temperature. The use of the hot water from the top of the tank, with any water used being replaced by cold water at the bottom. The collector then heats up the cold water again.

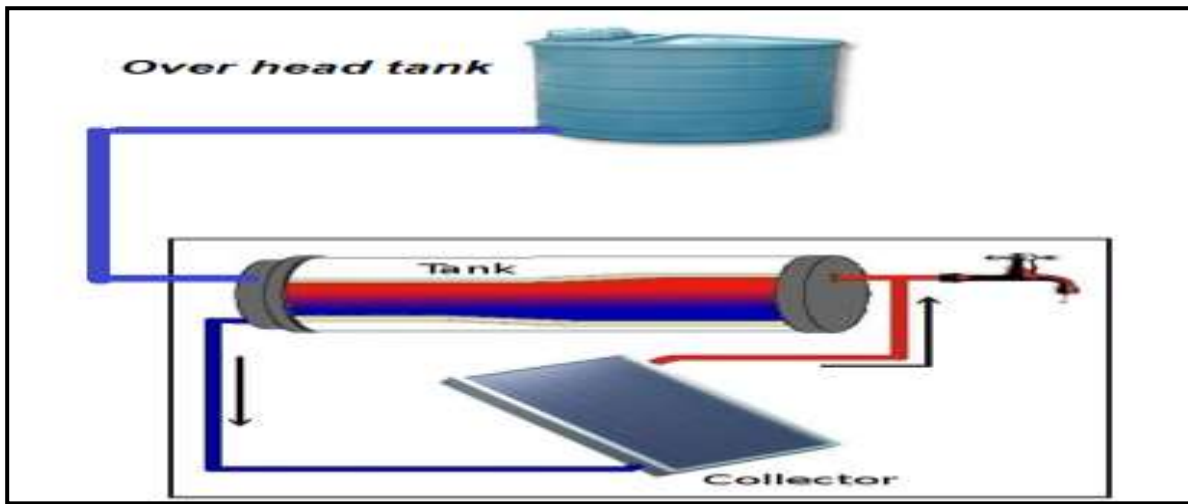


Figure 1. Thermosyphonong Principle

Figure 1 shows the thermo siphoning process. As shown, the overhead tanks are installed on the top so that the water can be easily circulated through the lower storage cylinder (SWG tank). As the water from outlet is tapped out, the water from the above tank keeps the cylinder full and running.

II. MATHEMATICAL MODELLING

To model the system in such a way that it fulfills the following requirements of the user

- It gives adequate amount of water for usage.
- The temperature of water acquired by using the system is usable comfortably.
- The warmth of water can be saved for a longer period of time.
- How different environmental factors affect the performance of the system.

To achieve these objectives and conditions, the following step by step calculation will assist to model it mathematically.

Starting the calculations by selecting the capacity of geyser's tank, for this it will be:

A. Capacity of SWG and Constraints Ruling It

The regular gas geysers installed in the houses start from capacity of 35 liters minimum and go up to 55 liters maximum. The main issue was to decide a capacity which is enough to fulfill the demand of a family for a whole day and to make sure that it will last for usage for a certain time period. The Volume of the tank of SWG decided for that reason is 75 liters as it is more than the maximum limit of the regular geysers and it can last for a longer time for a bigger family.

1. Deciding Length of Tank

The tank must be of a length which is not very enormous so that it does not affect the area where the whole system needs to be placed. It is also not very pleasant to see a long cylinder. Keeping these facts under consideration, the length selected is 1m, so that the cylinder does not take much of the area length and looks presentable.

2. Deciding the Radius

Now the length of the Geyser is decided so we have to compensate with the radius of the cylindrical tank so that it gives us the volume of 75 liters exactly. So the radius can be calculated by using volume constraints. A sketch of the tank is shown in figure 2 to clarify the dimensions.

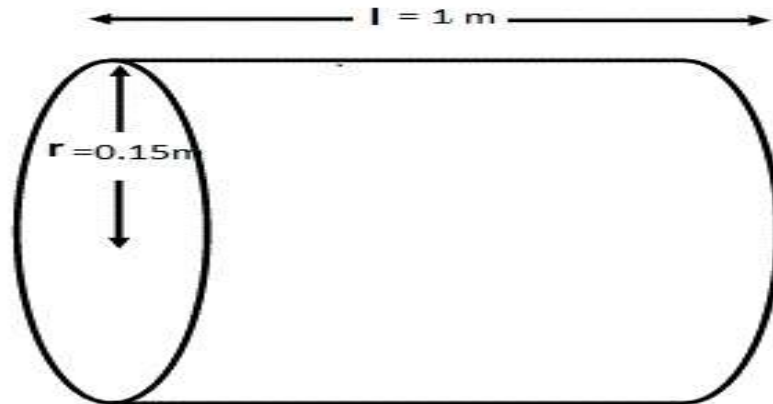


Figure 2. Cylindrical Tank Dimensions

Figure 2 shows the dimensions of the tank.

Now mathematically,

$$3. \text{ Volume of Tank } = V = \pi r^2 l \quad (1)$$

Rearranging Equation 4.1

$$r = (V/\pi l)^{1/2}$$

Putting

$$l = 1 \text{ m and}$$

$$V = 0.075 \text{ m}^3$$

$$r = 0.1524 \text{ m or } r = 6 \text{ inches}$$

B. Surface Area of Solar Collector and Constraints Governing It

The surface area of the collector is decided by mathematical modeling, but certain constraints (market and physical) govern the area optimization. Here the figure 3 will explain the constraints governing the SWG collector area.

Here the collector area is the absorber plate area and it depends upon the flow tubes which are of copper, the main constraint is the availability of copper, as copper tubes are available in some specific diameters and specific lengths, among which the price factor plays an important role, the 1 inch diameter pipe of 1 meter length is much more costly than the 0.88 inch diameter pipe of same length. So this is the most important constraint if economic resources are not strong. So, 15 tubes of 0.88 inch and 1 meter length are selected. But afterwards the theoretical values of results are also calculated with changing

diameter and lengths of the tubes or surface area. There is not much difference in the value of outlet water temperature by changing diameter; calculations are shown in the coming topic.

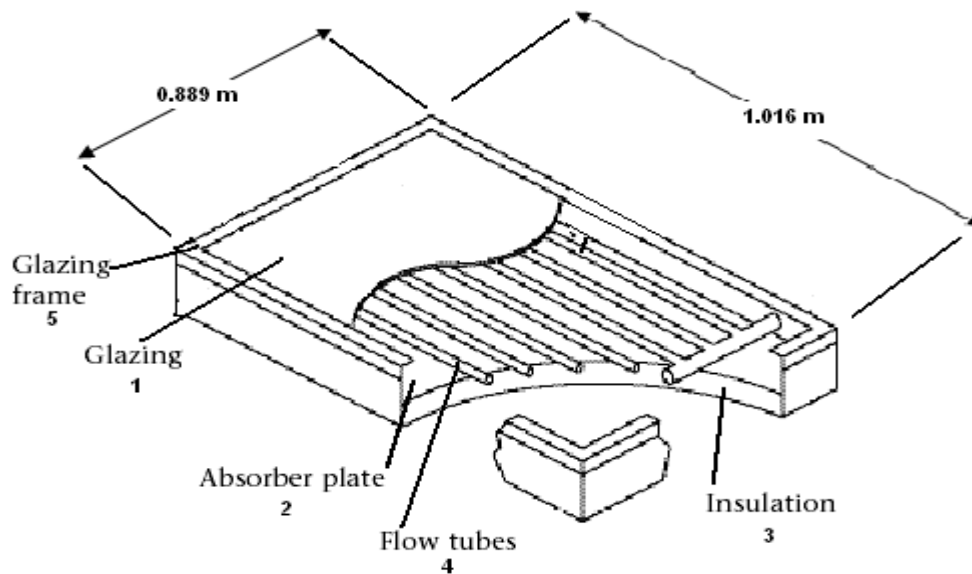


Figure 3. Model of the Solar Collector

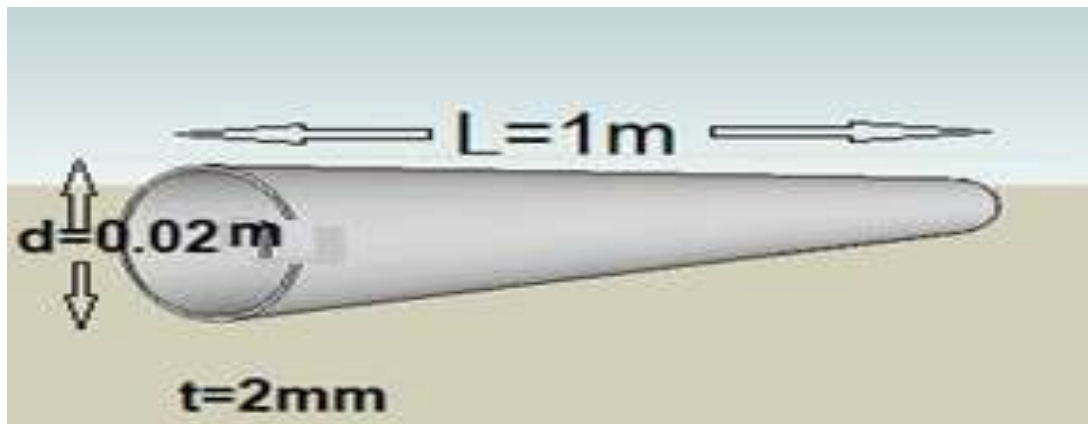


Figure 4. Pipes Dimensions

The figure 4 shows dimensions of the selected tubes. The width of the plate is set according to the centre to centre distance between copper tubes i-e 50mm, it is 0.903 m. So the overall area of the plate hence solar collector becomes

$$A = L \times w = 1 \times 0.903 = 0.903 \text{ m}^2.$$

C. Outlet Temperature, Efficiency and Different Factors Effecting Them ¹

The outlet temperature of the water and the overall system is an outcome of a very detailed process of calculating different factors and coefficients such as heat transfer coefficient, overall heat loss coefficient, fin effectiveness, as copper tubes are considered as circular fins and many other factors. All these factors are calculated using different Equations. The step by step calculation and algorithm is shown in this topic and an MS-Excel sheet is made for this algorithm to derive graphs by doing iterations in input values and other factors.

The table 1 shows all the data required for modeling the system. These values are used in the upcoming set of Equations.

Table 1 Data Required For Modeling the System

Data For The Required SWG System		
Specific Heat at Constant Pressure	$C_{p(\text{water})}$	4180 J/kg K
Thermal Conductivity of Water	K_{water}	0.637 W/m K
Thermal Conductivity of Sheet	K_{steel}	43 W/m K
Pie	π	3.1415
Center to center Spacing between Tubes	w	50 mm
Fin Effectiveness	F	Calculated ahead
Incidence Radiation	G	850 W/m ²
Inlet fluid temperature	T_{fi}	293 K
Ambient Temperature	T_a	288 K
Transmittivity of Glass	τ	0.85
Absorbance of the Paint	α	0.95
Overall Heat loss Coefficient from Plate to Absorber	U	8 W/m ² K
Volume of storage tank	V	0.075 m ³
Time taken for heating water	t	2 hrs
Density of water	ρ	1000 kg/m ³
Overall Mass Flow rate	m	0.010416 kg/sec
Outlet Fluid Temperature	T_{fo}	Calculated ahead
overall heat loss coefficient	U_L	8 W/m²K
Thickness of Sheet	t_{sb}	0.001 m
Inner Diameter of Tubes	D_i	0.0206 m
Outer Diameter of Tubes	D_o	0.0226 m
width of Sheet	W	0.889 m
length of Sheet	L	1.016 m
Area of Absorber plate	A_{so}	0.903 m ²

These Equation use all the value specified in the table 1

$$Q_f = F' A_{ab} \left[G \times \tau \times \alpha - U(T_{fm} - T_a) \right] \quad (2)$$

Radiation Absorbed by Fluid = $Q_f = 490.431$ W

$$T_{fm} = \frac{T_{fi} + T_{fo}}{2} \quad (3)$$

Fluid Mean Temperature = $T_{fm} = 30^\circ\text{C}$

Where; extra losses from fluid to plate can be calculated through the Equation (4)

$$F' = \frac{1/U_L}{W \left[\frac{1}{U_L [D + (W - D)F]} + \frac{\delta x_t}{\pi D K_t} + \frac{1}{\pi D_i h_{fi}} \right]} \quad (4)$$

Extra Losses from Fluid to Plate = $F' = 0.95039$

Fin Effectiveness is given in Equation (5)

$$F = \left\{ \frac{\tanh\left(\mathbf{m} \cdot \frac{\mathbf{W} - \mathbf{D}}{2}\right)}{\mathbf{m} \left(\frac{\mathbf{W} - \mathbf{D}}{2}\right)} \right\} \quad (5)$$

For convenience in above Equation m is just a variable it is calculated by using Equation below

$$\mathbf{m} = \sqrt{\frac{U}{K \times t_{ab}}}$$

Where

U = Overall heat loss coefficient from the absorber plate

K = Thermal conductivity of absorber plate

t_{ab} = Absorber Plate thickness

Putting values in above Equation

$$m = 13.6399$$

By putting " m " in Equation (5)

$$\mathbf{Fin\ Effectiveness} = \mathbf{F} = \mathbf{0.98852}$$

Here fin effectiveness is the copper tubes effectiveness as it allows nearly all the heat transfer to the water inside.

Now in order to find h_f it is important to know that whether the flow is laminar or turbulent, the Reynolds number is to be find, therefore the velocity of fluid in the tubes is found from the volumetric flow rate "q".

$$\text{As } q = V_{\text{tank}} / \text{time} = 0.075 / 7200 = 0.0000104 \text{ m}^3/\text{sec}$$

$$\text{Also } q = v \cdot A_{\text{tubes}} \quad (6)$$

Here "v" is the velocity of fluid and A is the area of tubes from which the fluid is passing

Putting values

$$0.0000104 = v \times 3.14 \times (0.0103)^2$$

this gives

$$\text{Velocity of fluid} = v = 0.00032 \text{ m/sec}$$

$$\text{Reynolds \#} = \text{Re} = \rho v D / \mu \quad (7)$$

$$\text{Here } \mu_{\text{water}} = 1.008 \times 10^{-3}$$

Reynolds number = Re = 7

As flow is laminar so from table I the value of Nusselt number for circular pipes and uniform heat flux is constant and given below

Nusselt number = Nu = 4.36

Calculation of Heat Transfer Coefficient for Hot Side Fluid h_{fi}

$$h_{fi} = \frac{N_u \times k_f}{D_i}$$

Putting values from table 1

Inside Convection of Pipe = $h_{fi} = 134.821 \text{ W/m}^2.\text{K}$

$$Q_f = m \cdot C_p (T_{fo} - T_{fi}) \tag{8}$$

Radiation Absorbed by Fluid = $Q_f = 490.431 \text{ W}$

Rearranging Equation (8)

$$T_{fo} = \frac{Q_f}{m \times C_p} + T_{fi} \tag{9}$$

Outlet Fluid Temperature = $T_{fo} = 38.7635 \text{ }^\circ\text{C}$

Equation (9) is the major Equation which gives the value of outlet temperature of water

Now for efficiency of the whole system is found by using Equation (10)

$$\eta = \frac{Q_f}{G \times A_{ab}} \tag{10}$$

Putting the values which have already been found

This gives the efficiency of the whole system

Efficiency = $\eta = 0.6388$

The outlet temperature and efficiency of the system for different parameters can be calculated by the algorithm created in the MS-Excel formula sheet.

D. Calculation for Insulation²

To heat water is one domain but to keep its heat conserved for a longer period of time so that it can be used at nights or in the evenings is a separate domain. For this purpose, insulation of the storage reservoir is required. The material selected for the insulation is glass wool as it is easily available and its thermal conductivity is very less i-e 0.05 W/m.k (from table 1).

1. Heat Loss without Insulation

First of all heat loss from the system without insulation is calculated as below

$$Q_{no\ ins} = \Delta T / \sum R_{th} \quad (11)$$

Here $Q_{no\ ins}$ = The heat loss from the storage tank without any insulation.

$$\Delta T = T_{inside\ tank} (T_1) - T_{outside\ environment} (T_2)$$

The inside water temperature is taken as 40°C and environment temperature is taken as 15°C for a mild winter night so

$$\Delta T = 25^\circ C$$

R_{th} = Thermal resistance of the thermal circuit.

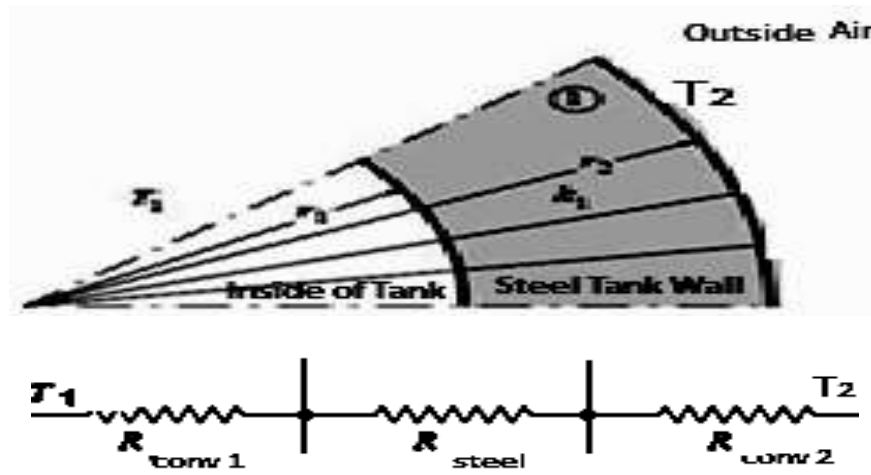


Figure 5. Thermal Circuit of Storage Tank

Figure 5. shows the cylinder thermal diagram and circuit of the tank

Here

$$r_1 = r_i \text{ (inner radius of the tank) } = 0.1504\text{m}$$

$$r_2 = r_o \text{ (outer radius of the tank) } = 0.1524\text{m}$$

$$k_1 = k_{steel} = 43 \text{ W/m} \cdot ^\circ C$$

R_{conv1} = Thermal resistance of inside water.

R_{cyl} or R_{steel} = Thermal resistance of steel tank.

R_{conv2} = Thermal resistance of outside environment (air).

Total thermal resistance of the circuit is given as

$$\sum R_{th} = R_{cyl} + R_{conv1} + R_{conv2} \quad (12)$$

$$R_{cyl} = \ell \ln (r_o/r_i) / 2 \pi k_{steel} \ell \quad (13)$$

$$R_{conv1} = 1/h_1 A_i \quad (14)$$

$$h_{water} = h_1 = 134 \text{ W/m}^2 \cdot ^\circ C \text{ (As calculated above)}$$

$$A_i = \text{Inner Area} = 2 \pi r_i \ell$$

Putting the value in Equation (13)

$$R_{\text{conv1}} = 1/h_1 2 \pi r_i \ell$$

Similarly

$$R_{\text{conv2}} = 1/h_2 A_o \tag{15}$$

h_2 or h_{air} 20 W/m² °C (h is the coefficient of convection and its value is 20 for non windy regions where velocity of air is not more than 10 m/s)

$$A_{\text{out}} = \text{Outer Area Of the Tank} = 2 \pi r_o \ell$$

$$R_{\text{conv2}} = 1/h_2 2 \pi r_o \ell$$

Putting respective values in Equation (13), (14) and (15)

$$R_{\text{cyl}} = 5.5 \times 10^{-5} \text{ } ^\circ\text{C/W}$$

$$R_{\text{conv1}} = 0.008 \text{ } ^\circ\text{C/W}$$

$$R_{\text{conv2}} = 0.058 \text{ } ^\circ\text{C/W}$$

Now putting these values in Equation (12) i-e

$$\sum R_{\text{th}} = 5.5 \times 10^{-5} + 0.008 + 0.058$$

$$\sum R_{\text{th}} = 0.0665 \text{ } ^\circ\text{C/W}$$

Now putting values of ΔT and $\sum R_{\text{th}}$ in Equation (11) i-e

$$Q_{\text{no ins}} = 25/0.0665$$

$$Q_{\text{no ins}} = 375 \text{ W}$$

This means that this amount heat energy from tank is being lost without insulation.

a. Prevention of Heat Loss

It is assumed that the glass wool insulation should minimize the heat loss to almost 95% from the tank.

Therefore

$$Q_{\text{ins}} = 5\% \text{ of } Q_{\text{no ins}} \tag{16}$$

$$Q_{\text{ins}} = 18 \text{ W}$$

This means that the heat loss will be reduced to 17.17 W from 343 W after insulation.

Now the thermal circuit will be different for insulated system.

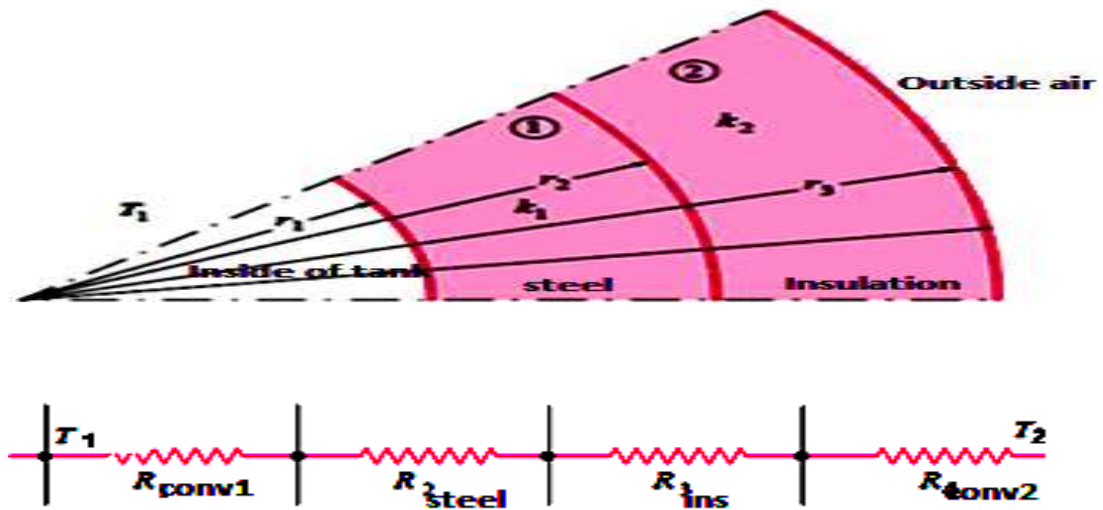


Figure 6. Thermal Circuit of the Insulated System

$$R_{ins} = \ell n (r_{ins}/r_2)/2 \pi k_{ins} \ell \tag{17}$$

$$\sum R_{th\ ins} = R_{conv1} + R_{cyl} + R_{ins} + R_{conv2} \tag{18}$$

$$\sum R_{th\ ins} = 0.014 + 5.5 \times 10^{-3} + \ell n (r_{ins}/0.1524)/ 0.279 + 1/h_{air} A_{ins}$$

Here the value of $A_{ins} = 2 \pi r_{ins} l$

So the Equation (18) becomes

$$\sum R_{th\ ins} = 0.014 + 5.5 \times 10^{-3} + \ell n (r_{ins}/0.1524)/ 0.279 + 1/h_{air} 2 \pi r_{ins} l$$

From Equation (11) we have

$$Q_{ins} = \Delta T / \sum R_{th\ ins}$$

Rearranging gives

$$\sum R_{th\ ins} = \Delta T / Q_{ins}$$

$$\sum R_{th\ ins} = 25/18 = 1.4$$

Putting this value of $\sum R_{th\ ins}$ in Equation (15)

$$1.4 = 0.014 + 5.5 \times 10^{-3} + \ell n (r_{ins}/0.1524)/ 0.279 + 1/h_{air} 2 \pi r_{ins} l$$

By iteration method the value of r_{ins} is found to be **0.2062 m** or **8.18 inches**

Now the insulation thickness is given by

$$t_{ins} = r_{ins} - r_{tank\ outer}$$

$$t_{ins} = 0.2077 - 0.1524$$

$t_{ins} = 0.05537 \text{ m}$ or $t = 2.18 \text{ inches}$

So, the insulation thickness of 2.18 inches is required for preventing 95% heat loss.

E. By Changing Surface Area of the Plate

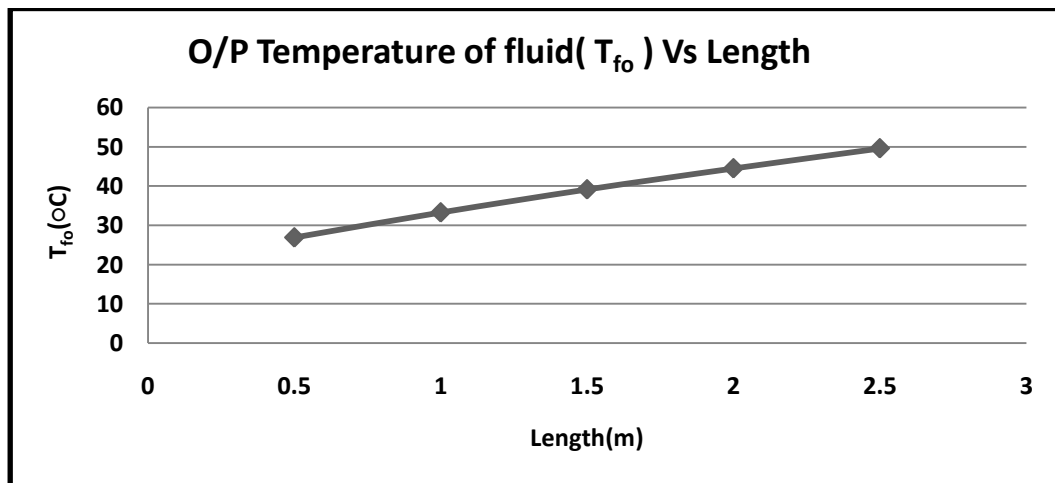
By changing the length or width of the collector absorbing plate the surface area is changed as

$$A = L \times w.$$

Here the length is varied and other parameters are kept constant in the MS-Excel formula sheet and its effect on outlet temperature and efficiency is calculated as shown in table 2.

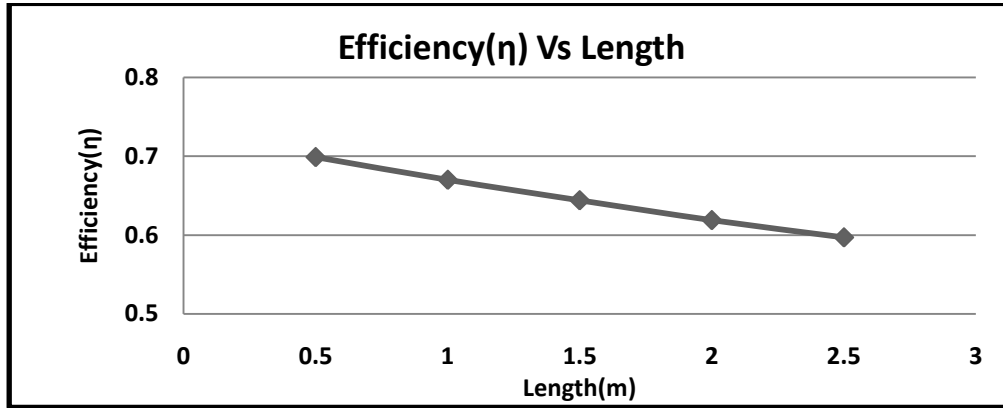
Table 2 Effect of Length on T_{out}

D	L	W	A	T_{fo}	H
0.0226	0.5	1.016	0.508	26.932	0.699
0.0226	1.0	1.016	1.016	33.301	0.670
0.0226	1.5	1.016	1.524	39.163	0.644
0.0226	2.0	1.016	2.032	44.517	0.619
0.0226	2.5	1.016	2.540	49.604	0.597



Graph 1. Effect of Length on T_{out}

Here the graph 1 is showing that with increasing length, the surface area is increasing and the heat absorbed is also increasing so, T_{out} is increasing.



Graph 2 Effect of Length on Efficiency

Graph 2 shows that increasing the length decreases the efficiency because increasing the length also increases the overall heat losses from fluid to environment and hence the collector does not give benefit according to its increased surface area.

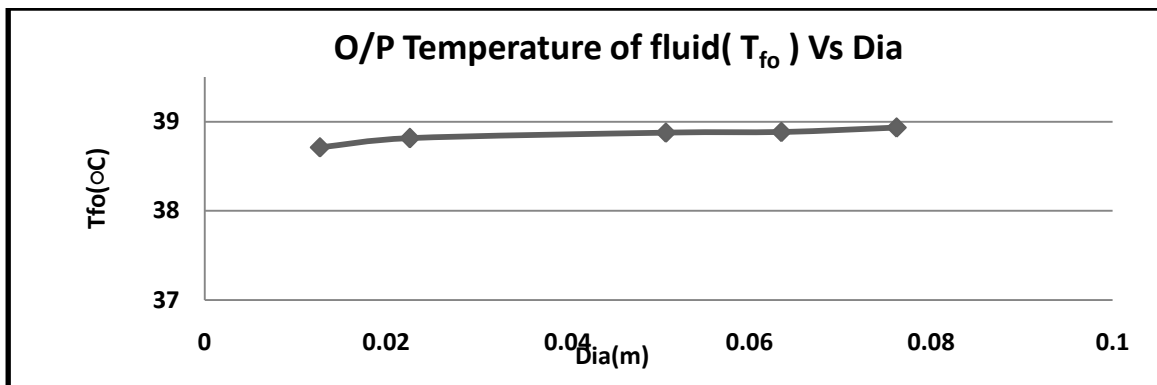
F. Effect of Diameter of Tubes

Now the diameter of tubes is varied and other terms are kept constant.

Table 3 Effect of Diameter of Tubes

D	L	W	A	Tfo	H
0.0127	0.889	1.016	0.903	38.712	0.664
0.0226	0.889	1.016	0.903	38.814	0.669
0.0508	0.889	1.016	0.903	38.877	0.673
0.0635	0.889	1.016	0.903	38.885	0.674
0.0762	0.889	1.016	0.903	38.935	0.676

Table 3 shows that there is a very minimal change in the outlet temperature, this is because the heat transferred is dependent on the plate length and width (its surface area). As copper tubes are acting as fins, therefore the selected diameter of copper tubes i-e 0.88 inch is optimum for the design specifications.



Graph 3 Effect of Tubes Diameter on T_{out}

Graph 3 shows the minimal changes in the outlet temperature.

III. Fabrication

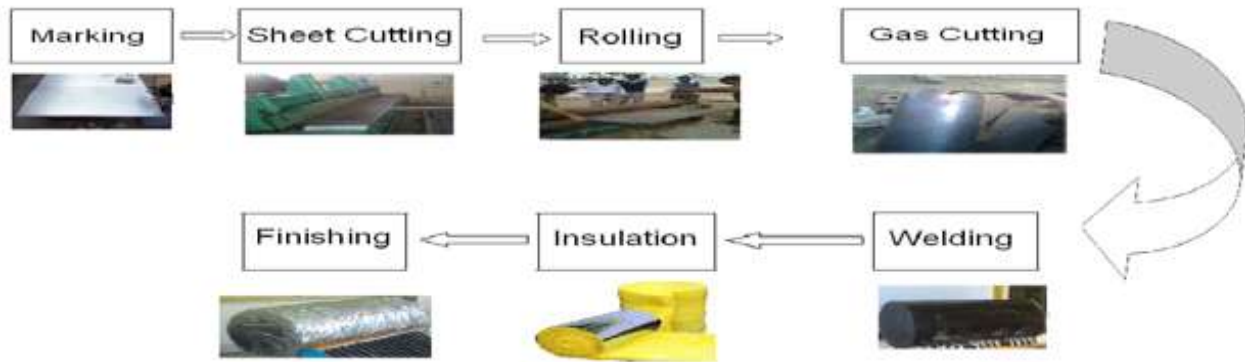


Figure 7. Fabrication Flow Chart of Solar Water Geyser

The figure shows the step by step fabrication process of the solar water geyser.

A. Modification in design

1. Black Metal Sheet for Increasing Surface Area of the Solar Collector

The box for the solar collector contains the copper pipes and the black sheet of 1 mm is attached to the copper pipes in such a way that there is an absolute contact of pipes to the metal sheet and the surface area for heat transfer is increased. In this way maximum heat is absorbed and transferred to the copper pipes by the surface area of the black sheet. As the box should be thermally insulated to avoid the heat loss from the box therefore an insulation of glass wool, two inches thick is also placed between sheet and box.

2. Glass for Green House Effect

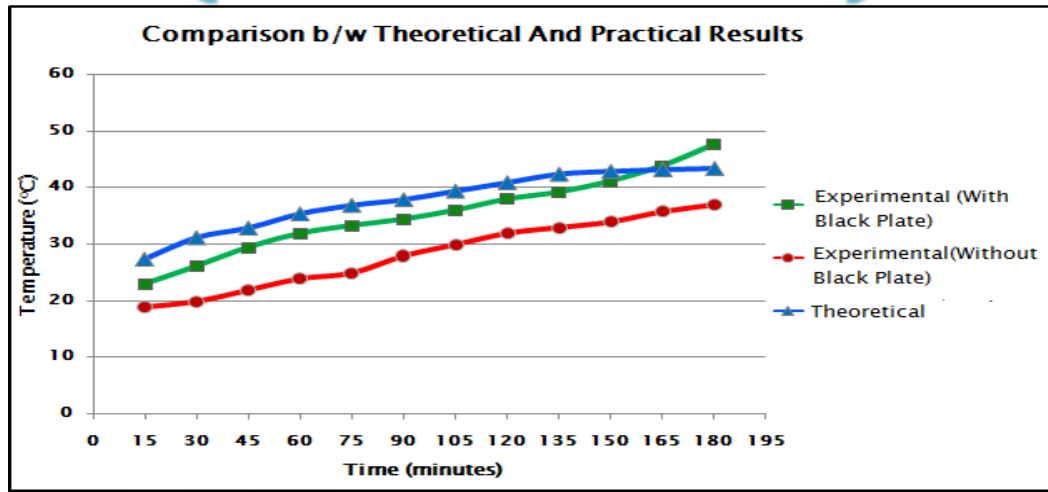
- The glass slot is left in the frame box for installing glass to reduce convectional heat losses and provide green house effect as discussed earlier.
- The glass of thickness 5mm and transmittivity 0.85 is placed in the slots. It stops the convection of cold air and allows the higher energy (short wavelength light) to enter the glass and as the heat radiations are absorbed by the black body inside, they emit the radiations which are now converted into low energy (longer wavelengths) radiations and the glass does not allow the low energy radiations to leave through it. In this way the radiations become entrapped in the collector box and heat up the water more efficiently. This is the basis of green house effect and the final fabricated model of the geyser is shown in the figure 8



Figure 8. Final Fabricated Model

IV. Results

A. Comparison b/w Theoretical And Practical Results (With/Without Abs Plate)



Graph 4. Comparison Between Theoretical And Practical Results

The graph 4 is showing the theoretical as well as experimental values of water outlet temperature. There is a slight difference because of different factors, sometimes the radiation exceeds the average assumed incidence radiation i-e 850 W/m^2 and sometimes the original value falls short of it.

V. Conclusions

The experimental run of the system and the practical data concludes the following:

- The system is well running and working very near to the theoretical results.
- This system can fulfill the demands for which it was modeled and fabricated.
- There are some different environmental parameters such as radiation flux ups and downs which can cause change in outlet temperature of the water.
- By using the black flat plate, the temperature change in experimental values is of a significant importance. T_{out} is increased and achieved in less time. This is because the black body absorbs more and hence green house effect is increased.

References

1. Solar Engineering of Thermal Processes (2nd edition) by william a.beckman
2. Cengel Y.A Heat Transfer

Note: Most of the analytical work is done manually just by the help of using the formulas given in the above reference books and the practical results validate the analytical results.